Pure Mathematics P3 Mark scheme

Question	Scheme	Marks
1	$9x^2 - 4 = (3x - 2)(3x + 2)$ at any stage	B1
	Eliminating the common factor of $(3x + 2)$ at any stage $\dfrac{2\cancel{(3x+2)}}{(3x-2)\cancel{(3x+2)}} = \dfrac{2}{3x-2}$	M1
	Use of a common denominator $\dfrac{2(3x+2)(3x+1)}{(9x^2-4)(3x+1)} - \dfrac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ **or** $\dfrac{2(3x+1)}{(3x-2)(3x+1)} - \dfrac{2(3x-2)}{(3x+1)(3x-2)}$	M1
	$\dfrac{6}{(3x-2)(3x+1)}$ **or** $\dfrac{6}{9x^2-3x-2}$	A1
		(4)
		(4 marks)

Notes:

B1: For factorising $9x^2 - 4 = (3x-2)(3x+2)$ using difference of two squares. It can be awarded at any stage of the answer but it must be scored on E pen as the first mark.

B1: For eliminating/cancelling out a factor of $(3x+2)$ at any stage of the answer.

M1: For combining two fractions to form a single fraction with a common denominator. Allow slips on the numerator but at least one must have been adapted. Condone invisible brackets. Accept two separate fractions with the same denominator as shown in the mark scheme. Amongst possible (incorrect) options scoring method marks are

$\dfrac{2(3x+2)}{(9x^2-4)(3x+1)} - \dfrac{2(9x^2-4)}{(9x^2-4)(3x+1)}$. Only one numerator adapted, separate fractions

$\dfrac{2\times 3x+1-2\times 3x-2}{(3x-2)(3x+1)}$ Invisible brackets, single fraction.

A1: $\dfrac{6}{(3x-2)(3x+1)}$

This is not a given answer so you can allow recovery from 'invisible' brackets.

Alternative

$$\frac{2(3x+2)}{(9x^2-4)} - \frac{2}{(3x+1)} = \frac{2(3x+2)(3x+1)-2(9x^2-4)}{(9x^2-4)(3x+1)} = \frac{18x+12}{(9x^2-4)(3x+1)}$$ has scored 0,0,1,0 so far

$$= \frac{6\cancel{(3x+2)}}{\cancel{(3x+2)}(3x-2)(3x+1)}$$ is now 1,1,1,0

$$= \frac{6}{(3x-2)(3x+1)}$$ and now 1,1,1,1

Question	Scheme	Marks
2(a)	$x^3 + 3x^2 + 4x - 12 = 0 \Rightarrow x^3 + 3x^2 = 12 - 4x$	
	$\Rightarrow x^2(x+3) = 12 - 4x$	M1
	$\Rightarrow x^2 = \dfrac{12-4x}{(x+3)}$ $\Rightarrow x = \sqrt{\dfrac{4(3-x)}{(x+3)}}$	dM1 A1*
		(3)
(b)	$x_1 = \sqrt{\left(\dfrac{4(3-1)}{(3+1)}\right)} = 1.41$	M1 A1
	awrt $x_2 = 1.20 \quad x_3 = 1.31$	A1
		(3)
(c)	Attempts $\text{f}(1.2725) = (+)0.00827... \quad \text{f}(1.2715) = -0.00821....$	M1
	Values correct with reason (change of sign with f(x) continuous) and conclusion ($\Rightarrow \alpha = 1.272$)	A1
		(2)
		(8 marks)

Notes:

(a)

M1: Moves from f(x) = 0, which may be implied by subsequent working, to $x^2(x \pm 3) = \pm 12 \pm 4x$ by separating terms and factorising in either order. No need to factorise rhs for this mark.

dM1: Divides by '(x+3)' term to make x^2 the subject, then takes square root. No need for rhs to be factorised at this stage.

A1*: CSO. This is a given solution. Do not allow sloppy algebra or notation with root on just numerator for instance. The 12−4x needs to have been factorised.

(b)

M1: An attempt to substitute $x_0 = 1$ into the iterative formula to calculate x_1. This can be awarded fo the sight of $\sqrt{\dfrac{4(3-1)}{(3+1)}}$, $\sqrt{\dfrac{8}{4}}$, $\sqrt{2}$ and even 1.4

A1: $x_1 = 1.41$. The subscript is not important. Mark as the first value found, $\sqrt{2}$ is A0

A1: $x_2 = \text{awrt } 1.20 \quad x_3 = \text{awrt } 1.31$. Mark as the second and third values found. Condone 1.2 for x_2

(c)

M1: Calculates f(1.2715) and f(1.2725), or **the** tighter interval with at least 1 correct to 1 sig fig rounded or truncated. Accept f(1.2715) = -0.008 1sf rounded or truncated. Also accept f(1.2715) = -0.01 2dp. Accept f(1.2725) = (+) 0.008 1sf rounded or truncated. Also accept f(1.2725) = (+)0.01 2dp

A1: Both values correct (see above),
A valid reason; Accept change of sign, or >0 <0, or f(1.2715) ×f(1.2725)<0
And a (minimal) conclusion; Accept hence root or α=1.272 or QED or $\square$

Question	Scheme	Marks
3(a)	Uses $-2(3-x)+5=\frac{1}{2}x+30$	M1
	Attempts to solve by multiplying out bracket, collect terms etc. $\frac{3}{2}x=31$	M1
	$x=\frac{62}{3}$ only	A1
		(3)
(b)	Makes the connection that there must be two intersections. Implied by either end point $k>5$ **or** $k\leqslant 11$	M1
	$5<k\leqslant 11$	A1
		(2)
		(5 marks)

Notes:

(a)

M1: Deduces that the solution to $\mathrm{f}(x)=\frac{1}{2}x+30$ can be found by solving

$-2(3-x)+5=\frac{1}{2}x+30$

M1: Correct method used to solve their equation. Multiplies out bracket/ collects like terms.

A1: $x=\frac{62}{3}$ only. Do not allow 20.6

(b)

M1: Deduces that two distinct roots occurs when $y=k$ intersects $y=\mathrm{f}(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k>5$ or $k\leqslant 11$

A1: Correct solution only $\{k: k\in\mathbb{R}, 5<k\leqslant 11\}$

Question	Scheme		Marks
4(i)	$\int \frac{1}{(2x-1)}\,dx = \frac{1}{2}\ln(2x-1)$		M1 A1
	$\int_5^{13} \frac{1}{(2x-1)}\,dx = \frac{1}{2}\ln 25 - \frac{1}{2}\ln 9 = \frac{1}{2}\ln\left(\frac{25}{9}\right)$		dM1
	$= \ln\left(\frac{5}{3}\right)$		A1
			(4)
(ii)	Integrates to give $\alpha\cos 2x + \beta\sec\frac{1}{3}x\{+c\}$ where $\alpha \neq 0, \beta \neq 0$ $\left[-\frac{1}{2}\cos 2x + 3\sec\frac{1}{3}x\{+c\}\right]$		M1
	$\left(-\frac{1}{2}\cos\left(2\times\frac{\pi}{2}\right)+3\sec\left(\frac{1}{3}\times\frac{\pi}{2}\right)\right)-\left(-\frac{1}{2}\cos(0)+3\sec(0)\right)$	Substitutes limits of 0 and $\frac{\pi}{2}$ and subtracts the correct way around	dM1
	$=2\sqrt{3}-2$		A1
			(3)
			(7 marks)

Notes:

(i)

M1: For $\int \frac{1}{(2x-1)}\,dx = k\ln(2x-1)$ where k is a constant.

A1: Correct integration $\int \frac{1}{(2x-1)}\,dx = \frac{1}{2}\ln(2x-1)$

dM1: Scored for substituting in the limits, subtracting and using correctly at least one log law.

You may see the subtraction law $k\ln 25 - k\ln 9 = k\ln\left(\frac{25}{9}\right)$ or the index law

$\frac{1}{2}\ln 25 - \frac{1}{2}\ln 9 = \ln 5 - \ln 3$

A1: cao $\ln\left(\frac{5}{3}\right)$

(ii)

M1: Integrates to a form $\alpha\cos 2x + \beta\sec\frac{1}{3}x\{+c\}$ where $\alpha \neq 0, \beta \neq 0$

dM1: Dependent upon the previous M1. It is scored for substituting limits of 0 and $\frac{\pi}{2}$ and subtracting the correct way around.

A1: cao $2\sqrt{3}-2$

Question	Scheme	Marks
5	$y=\dfrac{5x^2-10x+9}{(x-1)^2}$	
	Differentiates numerator to $10x-10$ and denominator to $2(x-1)$ o.e.	B1
	Uses the quotient rule $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(x-1)^2(10x-10)-\left(5x^2-10x+9\right)2(x-1)}{(x-1)^4}$	M1 A1
	Takes out a common factor from the numerator and cancels $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\cancel{(x-1)}\left\{(x-1)(10x-10)-\left(5x^2-10x+9\right)2\right\}}{(x-1)^{\cancel{4}3}}$	M1
	Simplifies the numerator by multiplying and collecting terms $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\left\{10x^2-20x+10-10x^2+20x-18\right\}}{(x-1)^3}$	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-8}{(x-1)^3}$	A1
		(6)
		(6 marks)

Notes:

B1: See scheme.

M1: Uses the quotient rule to reach a form $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(x-1)^2(Ax+B)-\left(5x^2-10x+9\right)(Cx+D)}{(x-1)^4}$ o.e.

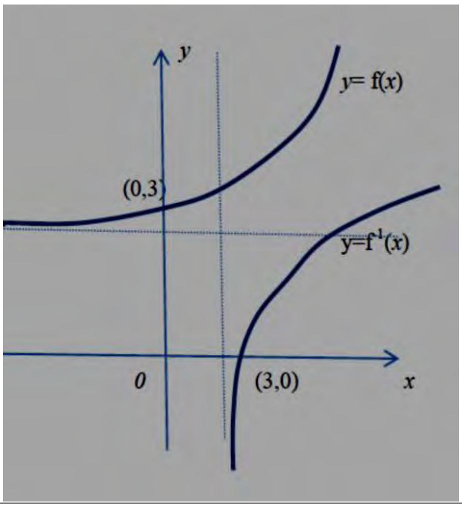
Alternatively uses the product rule to reach a for $\dfrac{\mathrm{d}y}{\mathrm{d}x}=(x-1)^{-2}(Ax+B)+\left(5x^2-10x+9\right)C(x-1)^{-3}$

A1: Fully correct $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ If the product rule is used $\dfrac{\mathrm{d}y}{\mathrm{d}x}=(x-1)^{-2}(10x-10)-\left(5x^2-10x+9\right)2(x-1)^{-3}$

M1: This is for using a correct method to reach a form $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{g}(x)}{(x-1)^3}$. See scheme when using the quotient rule. If the product rule is used it is for combining the terms using a common denominator.

M1: Scored for simplifying the numerator (By multiplying out and collecting terms).

A1: $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-8}{(x-1)^3}$

Question	Scheme	Marks
6(a)	$\mathrm{f}(x) > 2$	B1
		(1)
(b)	$\mathrm{fg}(x) = e^{\ln x} + 2, = x + 2$	M1 A1
		(2)
(c)	$e^{2x+3} + 2 = 6 \Rightarrow e^{2x+3} = 4$	M1 A1
	$\Rightarrow 2x + 3 = \ln 4$	
	$\Rightarrow x = \dfrac{\ln 4 - 3}{2}$ or $\ln 2 - \dfrac{3}{2}$	M1 A1
		(4)
(d)	Let $y = e^x + 2 \Rightarrow y - 2 = e^x \Rightarrow \ln(y-2) = x$	M1
	$\mathrm{f}^{-1}(x) = \ln(x-2),\ x > 2$	A1 B1ft
		(3)
(e)	 Shape for $\mathrm{f}(x)$	B1
	(0, 3)	B1
	Shape for $\mathrm{f}^{-1}(x)$	B1
	(3, 0)	B1
		(4)
		(14 marks)

Notes:

(a)

B1: Range of f(x)>2. Accept y>2, (2,∞), f>2, as well as 'range is the set of numbers bigger than 2' but **don't accept** x >2

(b)

M1: For applying the correct order of operations. Look for $e^{\ln x} + 2$. Note that $\ln e^x + 2$ is M0

A1: Simplifies $e^{\ln x} + 2$ to $x + 2$. Just the answer is acceptable for both marks.

(c)

M1: Starts with $e^{2x+3} + 2 = 6$ and proceeds to $e^{2x+3} = ...$

A1: $e^{2x+3} = 4$

M1: Takes ln's both sides, $2x + 3 = \ln ..$ and proceeds to x=….

Question 6 notes *continued*	
A1:	$x = \dfrac{\ln 4 - 3}{2}$ oe. eg $\ln 2 - \dfrac{3}{2}$ Remember to isw any incorrect working after a correct answer.
(d)	
M1:	Starts with $y = e^x + 2$ or $x = e^y + 2$ and attempts to change the subject. All ln work must be correct. The 2 must be dealt with first. Eg. $y = e^x + 2 \Rightarrow \ln y = x + \ln 2 \Rightarrow x = \ln y - \ln 2$ is M0.
A1:	$\mathrm{f}^{-1}(x) = \ln(x-2)$ or $y = \ln(x-2)$ or $y = \ln\lvert x-2\rvert$ There must be some form of bracket.
B1ft:	Either $x > 2$, or follow through on their answer to part (a), provided that it wasn't $y \in \Re$ Do not accept $y > 2$ or $\mathrm{f}^{-1}(x) > 2$.
(e)	
B1:	Shape for $y = e^x$. The graph should only lie in quadrants 1 and 2. It should start out with a gradient that is approx. 0 above the x axis in quadrant 2 and increase in gradient as it moves into quadrant 1. You should not see a minimum point on the graph.
B1:	(0, 3) lies on the curve. Accept 3 written on the y axis as long as the point lies on the curve.
B1:	Shape for $y = \ln x$. The graph should only lie in quadrants 4 and 1. It should start out with gradient that is approx. infinite to the right of the y axis in quadrant 4 and decrease in gradient as it moves into quadrant 1. You should not see a maximum point. Also with hold this mark if it intersects $y = e^x$.
B1:	(3, 0) lies on the curve. Accept 3 written on the x axis as long as the point lies on the curve.

Question	Scheme	Marks
7(a)	$p = 4\pi^2$ **or** $(2\pi)^2$	B1
		(1)
(b)	$x = (4y - \sin 2y)^2 \Rightarrow \dfrac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$	M1 A1
	Sub $y = \dfrac{\pi}{2} \Rightarrow \dfrac{dx}{dy} = 24\pi \quad (= 75.4)$ OR $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{24\pi} (= 0.013)$	M1
	Equation of tangent $y - \dfrac{\pi}{2} = \dfrac{1}{24\pi}\ x - 4\pi^2$	M1
	Using $y - \dfrac{\pi}{2} = \dfrac{1}{24\pi}\ x - 4\pi^2$ with $x = 0 \Rightarrow y = \dfrac{\pi}{3}$ **cso**	M1 A1
		(6)
	Alternative I for first two marks	
	$x = (4y - \sin 2y)^2 \Rightarrow x^{0.5} = 4y - \sin 2y$ $\Rightarrow 0.5x^{-0.5}\dfrac{dx}{dy} = 4 - 2\cos 2y$	M1A1
	Alternative II for first two marks	
	$x = \left(16y^2 - 8y\sin 2y + \sin^2 2y\right)$ $\Rightarrow 1 = 32y\dfrac{dy}{dx} - 8\sin 2y\dfrac{dy}{dx} - 16y\cos 2y\dfrac{dy}{dx} + 4\sin 2y\cos 2y\dfrac{dy}{dx}$ **Or** $1dx = 32y\,dy - 8\sin 2y\,dy - 16y\cos 2y\,dy + 4\sin 2y\cos 2y\,dy$	M1A1
		(7 marks)

Notes:

(a)

B1: $p = 4\pi^2$ or exact equivalent $2\pi\ ^2$. Also allow $x = 4\pi^2$

(b)

M1: Uses the chain rule of differentiation to get a form $A(4y - \sin 2y)(B \pm C\cos 2y)$, $\quad A, B, C \neq 0$ on the right hand side.

Alternatively attempts to expand and then differentiate using product rule and chain rule to a form $x = \left(16y^2 - 8y\sin 2y + \sin^2 2y\right) \Rightarrow \dfrac{dx}{dy} = Py \pm Q\sin 2y \pm Ry\cos 2y \pm S\sin 2y\cos 2y \quad P, Q, R, S \neq 0$

A second method is to take the square root first. To score the method look for a differentiated expression of the form $Px^{-0.5}... = 4 - Q\cos 2y$

A third method is to multiply out and use implicit differentiation. Look for the correct terms, condoning errors on just the constants.

Question 7 notes *continued*

A1: $\frac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$ or $\frac{dy}{dx} = \frac{1}{2(4y - \sin 2y)(4 - 2\cos 2y)}$ with both sides correct. The lhs may be seen elsewhere if clearly linked to the rhs. In the alternative $\frac{dx}{dy} = 32y - 8\sin 2y - 16y\cos 2y + 4\sin 2y\cos 2y$

M1: Sub $y = \frac{\pi}{2}$ into their $\frac{dx}{dy}$ or inverted $\frac{dx}{dy}$. Evidence could be minimal, eg $y = \frac{\pi}{2} \Rightarrow \frac{dx}{dy} = ...$

It is not dependent upon the previous M1 but it must be a changed $x = (4y - \sin 2y)^2$

M1: Score for a correct method for finding the equation of the tangent at $\left('4\pi^2', \frac{\pi}{2}\right)$.

Allow for $y - \frac{\pi}{2} = \frac{1}{\text{their numerical } {}^{dx}/_{dy}} \; x - \text{their } 4\pi^2$

Allow for $\left(y - \frac{\pi}{2}\right) \times \text{their numerical } {}^{dx}/_{dy} = x - \text{their } 4\pi^2$

Even allow for $y - \frac{\pi}{2} = \frac{1}{\text{their numerical } {}^{dx}/_{dy}} \; x - p$

It is possible to score this by stating the equation $y = \frac{1}{24\pi}x + c$ as long as $\left('4\pi^2', \frac{\pi}{2}\right)$ is used in a subsequent line.

M1: Score for writing their equation in the form $y = mx + c$ and stating the value of 'c'

or setting $x = 0$ in their $y - \frac{\pi}{2} = \frac{1}{24\pi} \; x - 4\pi^2$ and solving for y.

Alternatively using the gradient of the line segment AP = gradient of tangent.

Look for $\frac{\frac{\pi}{2} - y}{4\pi^2} = \frac{1}{24\pi} \Rightarrow y = ..$ Such a method scores the previous M mark as well.

At this stage all of the constants must be numerical. It is not dependent and it is possible to score this using the "incorrect" gradient.

A1: **cso** $y = \frac{\pi}{3}$. You do not have to see $\left(0, \frac{\pi}{3}\right)$

Question	Scheme	Marks
8(a)	$N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b$	M1
	$\Rightarrow \log_{10} N = \log_{10} a + b\log_{10} T$ so $m = b$ **and** $c = \log_{10} a$	A1
		(2)
(b)	Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** b = gradient	M1
	Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** b = gradient	M1
	Uses $T = 3$ in $N = aT^b$ with their a and b	M1
	Number of microbes ≈ 800	A1
		(4)
(c)	States that 'a' is the number of microbes 1 day after the start of the experiment.	B1
		(1)
		(7 marks)

Notes:

(a)

M1: Takes $\log_{10}$'s of both sides and attempts to use the addition law. Condone $\log = \log_{10}$ for this mark.

A1: Proceeds correctly to $\log_{10} N = \log_{10} a + b\log_{10} T$ **and** states $m = b$ and $c = \log_{10} a$

(b) **Way One: Main scheme**

M1: For attempting to use the graph to find either a or b using $a = 10^{\text{intercept}}$ **or** b = gradient. This may be implied by $a = 10^{1.75 \text{ to } 1.85}$ or b = 2.27 to 2.33

M1: For attempting to use the graph to find BOTH a and b (See previous M1)

M1: Uses $T = 3$ in $N = aT^b$ with their a and b

A1: Number of microbes ≈ 800

Way Two: Alternative using line of best fit techniques.

M1: For $\log_{10} 3 \approx 0.48$ and using the graph to find $\log_{10} N$

M1: For using the graph to find $\log_{10} N$ (FYI $\log_{10} N \approx 2.9$)

M1: For $\log_{10} N = k \Rightarrow N = 10^k$

A1: Number of microbes ≈ 800

(c)

B1: See scheme.

Question	Scheme	Marks
9(a)	$\sec 2A + \tan 2A = \dfrac{1}{\cos 2A} + \dfrac{\sin 2A}{\cos 2A}$	B1
	$= \dfrac{1+\sin 2A}{\cos 2A}$	M1
	$= \dfrac{1+2\sin A\cos A}{\cos^2 A - \sin^2 A}$	M1
	$= \dfrac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$	
	$= \dfrac{(\cos A+\sin A)(\cos A+\sin A)}{(\cos A+\sin A)(\cos A-\sin A)}$	M1
	$= \dfrac{\cos A+\sin A}{\cos A-\sin A}$	A1*
		(5)
(b)	$\sec 2\theta + \tan 2\theta = \dfrac{1}{2} \Rightarrow \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \dfrac{1}{2}$	
	$\Rightarrow 2\cos\theta + 2\sin\theta = \cos\theta - \sin\theta$	
	$\Rightarrow \tan\theta = -\dfrac{1}{3}$	M1 A1
	$\Rightarrow \theta =$ awrt 2.820, 5.961	dM1 A1
		(4)
		(9 marks)

Notes:

(a)

B1: A correct identity for $\sec 2A = \dfrac{1}{\cos 2A}$ **or** $\tan 2A = \dfrac{\sin 2A}{\cos 2A}$.

It need not be in the proof and it could be implied by the sight of $\sec 2A = \dfrac{1}{\cos^2 A - \sin^2 A}$

M1: For setting their expression as a single fraction. The denominator must be correct for their fractions and at least two terms on the numerator.

This is usually scored for $\dfrac{1+\cos 2A\tan 2A}{\cos 2A}$ or $\dfrac{1+\sin 2A}{\cos 2A}$

M1: For getting an expression in just sin A and cos A by using the double angle identities $\sin 2A = 2\sin A\cos A$ and $\cos 2A = \cos^2 A - \sin^2 A$, $2\cos^2 A - 1$ or $1-2\sin^2 A$.

Alternatively for getting an expression in just sin A and cos A by using the double angle identities $\sin 2A = 2\sin A\cos A$ and $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ with $\tan A = \dfrac{\sin A}{\cos A}$.

For example $= \dfrac{1}{\cos^2 A - \sin^2 A} + \dfrac{2\sin A/\cos A}{1 - \sin^2 A/\cos^2 A}$ is B1M0M1 so far

Question 9 notes *continued*	
M1:	In the main scheme it is for replacing 1 by $\cos^2 A+\sin^2 A$ **and** factorising both numerator and denominator.
A1*:	Cancelling to produce given answer with no errors. Allow a consistent use of another variable such as θ, but mixing up variables will lose the A1*.
(b)	
M1:	For using part (a), cross multiplying, dividing by $\cos\theta$ to reach $\tan\theta = k$ Condone $\tan 2\theta = k$ for this mark only.
A1:	$\tan\theta = -\frac{1}{3}$
dM1:	Scored for $\tan\theta = k$ leading to at least one value (with 1 dp accuracy) for θ between 0 and 2π. You may have to use a calculator to check. Allow answers in degrees for this mark.
A1:	$\theta =$ awrt 2.820, 5.961 with no extra solutions within the range. Condone 2.82 for 2.820. You may condone different/ mixed variables in part (b)

Question	Scheme	Marks
10(a)	Subs $D=15$ and $t=4$ $\quad x = 15\mathrm{e}^{-0.2\times4} = 6.740$ (mg)	M1 A1
		(2)
(b)	$15\mathrm{e}^{-0.2\times7} + 15\mathrm{e}^{-0.2\times2} = 13.754$ (mg)	M1 A1*
		(2)
(c)	$15\mathrm{e}^{-0.2\times T} + 15\mathrm{e}^{-0.2\times(T+5)} = 7.5$	M1
	$15\mathrm{e}^{-0.2\times T} + 15\mathrm{e}^{-0.2\times T}\mathrm{e}^{-1} = 7.5$	
	$15\mathrm{e}^{-0.2\times T}(1+\mathrm{e}^{-1}) = 7.5 \Rightarrow \mathrm{e}^{-0.2\times T} = \dfrac{7.5}{15\left(1+\mathrm{e}^{-1}\right)}$	dM1
	$\mathrm{T} = -5\ln\left(\dfrac{7.5}{15\left(1+\mathrm{e}^{-1}\right)}\right) = 5\ln\left(2+\dfrac{2}{\mathrm{e}}\right)$	A1 A1
		(4)
		(8 marks)

Notes:

(a)

M1: Attempts to substitute both $D=15$ and $t=4$ in $x = D\mathrm{e}^{-0.2t}$. It can be implied by sight of $15\mathrm{e}^{-0.8}$, $15\mathrm{e}^{-0.2\times4}$ or awrt 6.7. Condone slips on the power. Eg you may see -0.02

A1: Cao. 6.740 (mg) Note that 6.74 (mg) is A0

(b)

M1: Attempt to find the sum of two expressions with $D=15$ in both terms with t values of 2 and 7. Evidence would be $15\mathrm{e}^{-0.2\times7} + 15\mathrm{e}^{-0.2\times2}$ or similar expressions such as $\left(15\mathrm{e}^{-1}+15\right)\mathrm{e}^{-0.2\times2}$. Award for the sight of the two numbers awrt **3.70** and awrt **10.05**, followed by their total awrt **13.75**. Alternatively finds the amount after 5 hours, $15\mathrm{e}^{-1}$ = awrt **5.52** adds the second dose = **15** to get a total of awrt **20.52** then multiplies this by $\mathrm{e}^{-0.4}$ to get awrt **13.75**. Sight of $5.52+15=20.52 \rightarrow 13.75$ is fine.

A1*: Cso so both the expression $15\mathrm{e}^{-0.2\times7} + 15\mathrm{e}^{-0.2\times2}$ and $13.754(mg)$ are required

Alternatively both the expression $\left(15\mathrm{e}^{-0.2\times5}+15\right)\times\mathrm{e}^{-0.2\times2}$ and 13.754 (*mg*) are required.

Sight of just the numbers is not enough for the A1*

(c)

M1: Attempts to write down a correct equation involving T or t. Accept with or without correct bracketing Eg. accept $15\mathrm{e}^{-0.2\times T} + 15\mathrm{e}^{-0.2\times(T\pm5)} = 7.5$ or similar equations $\left(15\mathrm{e}^{-1}+15\right)\mathrm{e}^{-0.2\times T} = 7.5$

dM1: Attempts to solve their equation, dependent upon the previous mark, by proceeding to $\mathrm{e}^{-0.2\times T} = ..$An attempt should involve an attempt at the index law $x^{m+n} = x^m \times x^n$ and taking out a factor of $\mathrm{e}^{-0.2\times T}$ Also score for candidates who make $\mathrm{e}^{+0.2\times T}$ the subject using the same criteria.

Question 10 notes *continued*
A1: Any correct form of the answer, for example, $-5\ln\left(\dfrac{7.5}{15\left(1+e^{-1}\right)}\right)$ **A1:** Cso. $T=5\ln\left(2+\dfrac{2}{e}\right)$ Condone *t* appearing for *T* throughout this question.
(c) **Alternative 1** 1st Mark (Method): $15e^{-0.2\times T}+\text{awrt }5.52e^{-0.2\times T}=7.5\Rightarrow e^{-0.2\times T}=\text{awrt }0.37$ 2nd Mark (Accuracy): $\text{T=-5ln}\left(\text{awrt }0.37\right)$ or awrt 5.03 or $\text{T=-5ln}\left(\dfrac{7.5}{\text{awrt }20.52}\right)$ **Alternative 2** 1st Mark (Method): $13.754e^{-0.2\times T}=7.5\Rightarrow T=-5\ln\left(\dfrac{7.5}{13.754}\right)$ or equivalent such as 3.03 2nd Mark (Accuracy): 3.03 + 2 = 5.03 Allow $-5\ln\left(\dfrac{7.5}{13.754}\right)+2$ **Alternative 3 (by trial and improvement)** 1st Mark (Method): $15e^{-0.2\times 5}+15e^{-0.2\times 10}=7.55$ or $15e^{-0.2\times 5.1}+15e^{-0.2\times 10.1}=7.40$ or any value between. 2nd Mark (Accuracy): Answer *T* =5.03.